Two Stream Instability Benchmark with ORBIT

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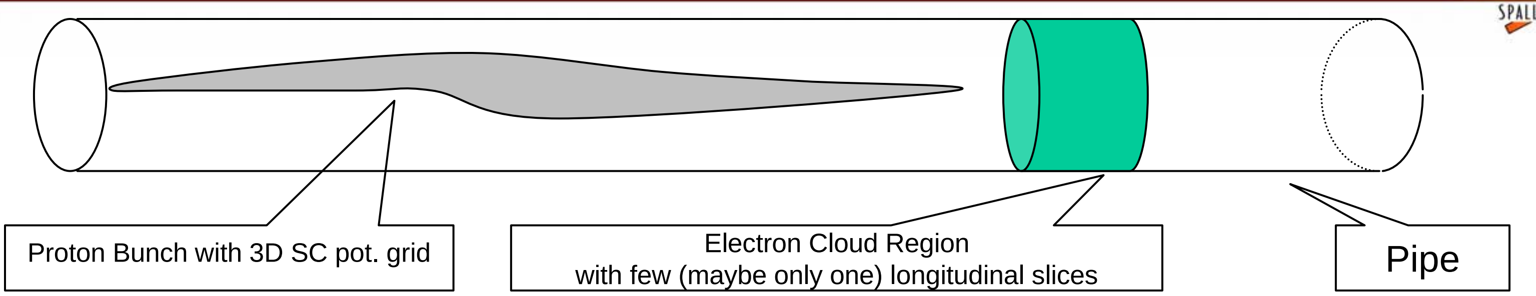
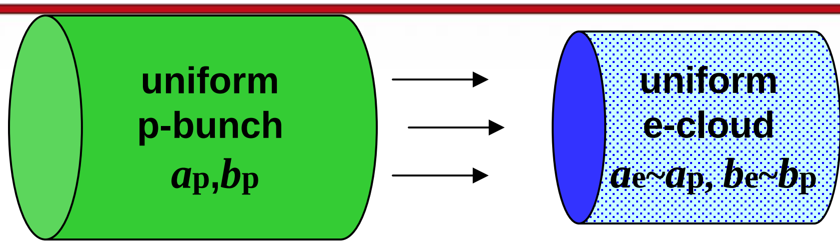
Outline



- Benchmark of instability for two stream model
  - Analytically solvable Electron Cloud Model
  - Two Stream Model in ORBIT
  - Instability and growth rate

- Estimation of computational requirements for PSR bunched beam case

Analytically Solvable Electron Cloud Model



Ref: D. Neuffer *et. al. NIM* **A321** p1 (1992)

With assuming harmonic oscillation in both centroid motion

$$y_{p,c} = A_p Exp[i(n\theta - \omega t)], \quad y_{e,c} = A_e Exp[i(n\theta - \omega t)]$$

n = longitudinal harmonic of ep mode

the equations of motion under no frequency spread lead

dispersion relation

$$A_e / A_p = \frac{\omega_e^2}{\omega_e^2 - \omega^2}, \qquad (\omega_e^2 - \omega^2)\{\omega_\beta^2 + \omega_p^2 - (n\omega_0 - \omega)^2\} = \omega_e^2 \omega_p^2$$

ep freq. (ω), betatron freq. (ω_β), rev. freq. (ω_0), ep freq. (ω)

$$\omega_{p,V}^2 = \frac{4\lambda_e r_p c^2}{\gamma b_e (a_e + b_e)}, \quad \omega_{e,V}^2 = \frac{4\lambda_p r_e c^2}{b_p (a_p + b_p)}$$

The relation is valid under **linear force** inside the streams

The dispersion relation has complex solutions (instability) near $\omega \sim \omega_e$ and $\omega \sim (n\omega_0 - \omega_\beta)$, slow wave, and satisfies the threshold condition:

$$\omega_p \gtrsim \sqrt{\omega_\beta / \omega_e}\, |n\omega_0 - \omega_e - \omega_\beta| = \omega_0 \sqrt{Q_\beta / Q_e}\, |n - Q_e - Q_\beta| \quad ; Q_e \equiv \omega_e / \omega_0 \; ; Q_\beta \equiv \omega_\beta / \omega_0$$

Two Stream Model in ORBIT

To study the two stream model in ORBIT, we use SNS parameters

$$a_e = b_e = a_p = b_p = 30\,\text{mm},\ 1\,\text{GeV protonbeam},\ \text{betatron tune } Q_x = Q_y = 6.2$$

$$\omega_0 = 2\pi/T = 6.646[\mu s^{-1}],\ \lambda_p = \tfrac{1.5*10^{14}}{248\text{m}*0.65}*(Bunchfactor = 2.5) = 2.326*10^{12}\,[\text{m}^{-1}]$$

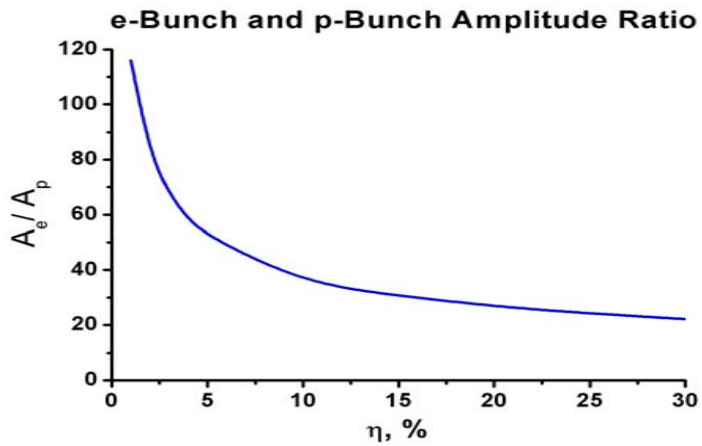
$$Q_e = \omega_e/\omega_0 = 172.171$$

$$Q_p = \omega_p/\omega_0 = 2.79616\sqrt{\eta} \quad ;\eta = \lambda_e/\lambda_p = \text{neutralization factor}$$

which is most unstable at the longitudinal harmonic number n = 178.
For sufficient electron cloud, exceeding the threshold, the dispersion relation for n = 178 has a growth mode as one of 4 roots of ω:

$$\omega_2/\omega_0 = 171.961 - 0.716i\,, \quad \left|A_e/A_p\right|_{\omega_2} = 116.1 \qquad \text{for } \eta = 0.01$$

So, if we initialize the electron cloud and proton beam as slow waves with n=178 modulation and proper phase relationship, we can expect EC centroid oscillation to grow.



Two stream model in ORBIT, cont.

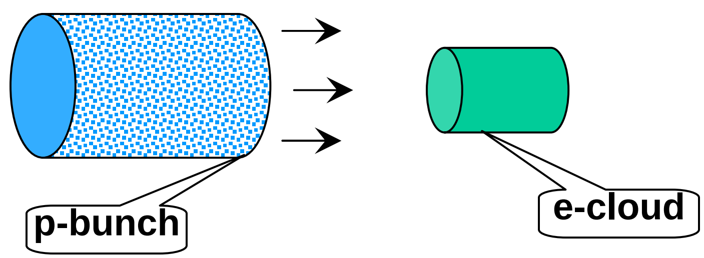
To reduce the calculation time, we adopt the periodic structure of L=248m/178=1.393m having 20 longitudinal nodes. $N_p = \lambda_p L = 3.241\times 10^{12}$

Initial proton bunch

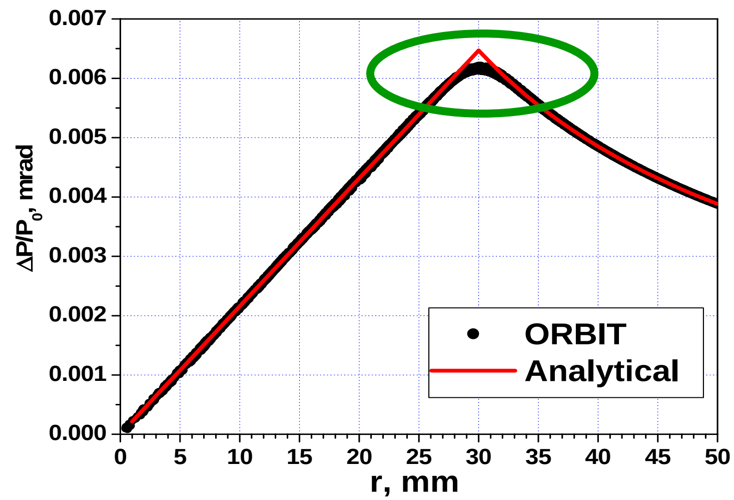
- KV distribution (R_p=30mm) –needs **very** (32 points) symmetric structure
- 0.01mm centroid modulation (slow wave) in vertical direction
- more than 400,000 macroprotons to satisfy at least 10 particles/grid-cell

Initial electron cloud

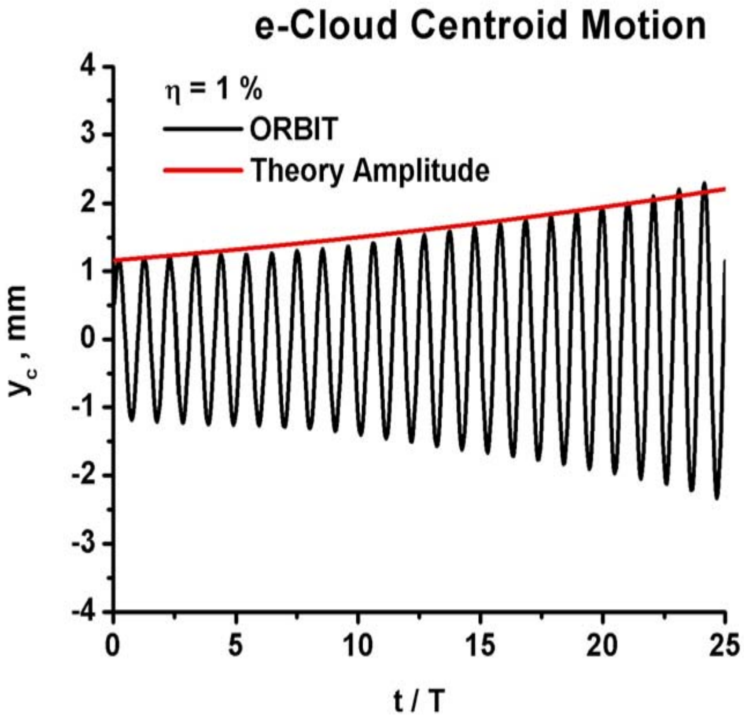
- KV distribution (R_e=26mm) –needs to receive **linear** force inside p-bunch
- 400,000 macroelectrons with $\lambda_e = \eta \left(\frac{R_e}{R_p}\right)^2 \lambda_p$
- $\left(A_e/A_p\right)_{\eta,\,\text{growth mode}} \times 0.01\text{mm}$ centroid modulation in vertical direction

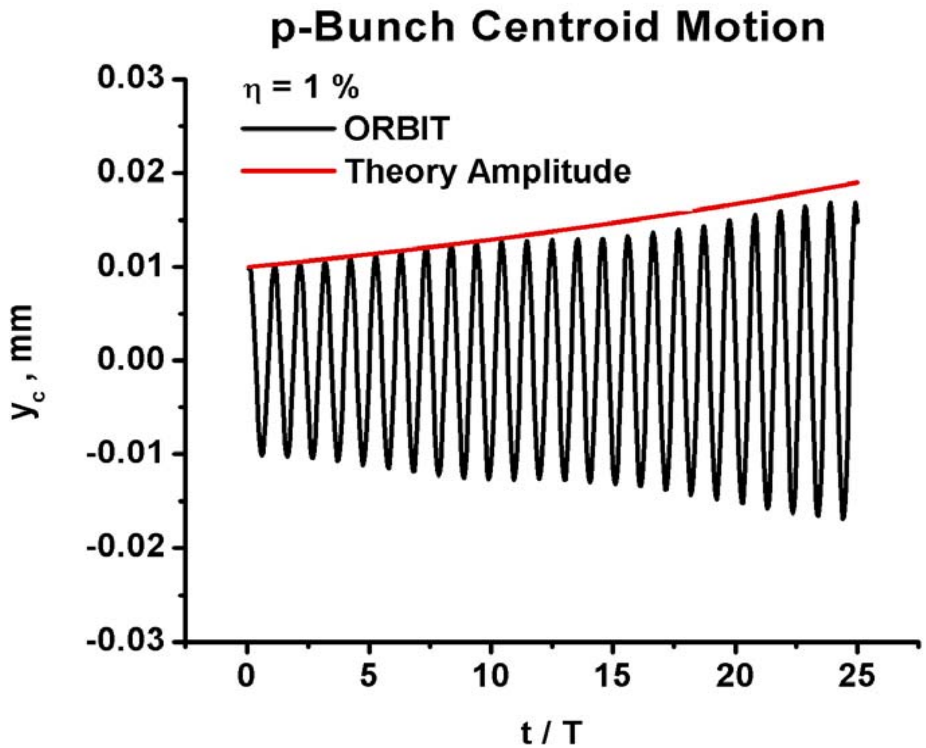


The change in the transverse momentum of protons is in perfect agreement with analytic calculations except for the round shoulder



Two stream benchmark (ORBIT Simulation)

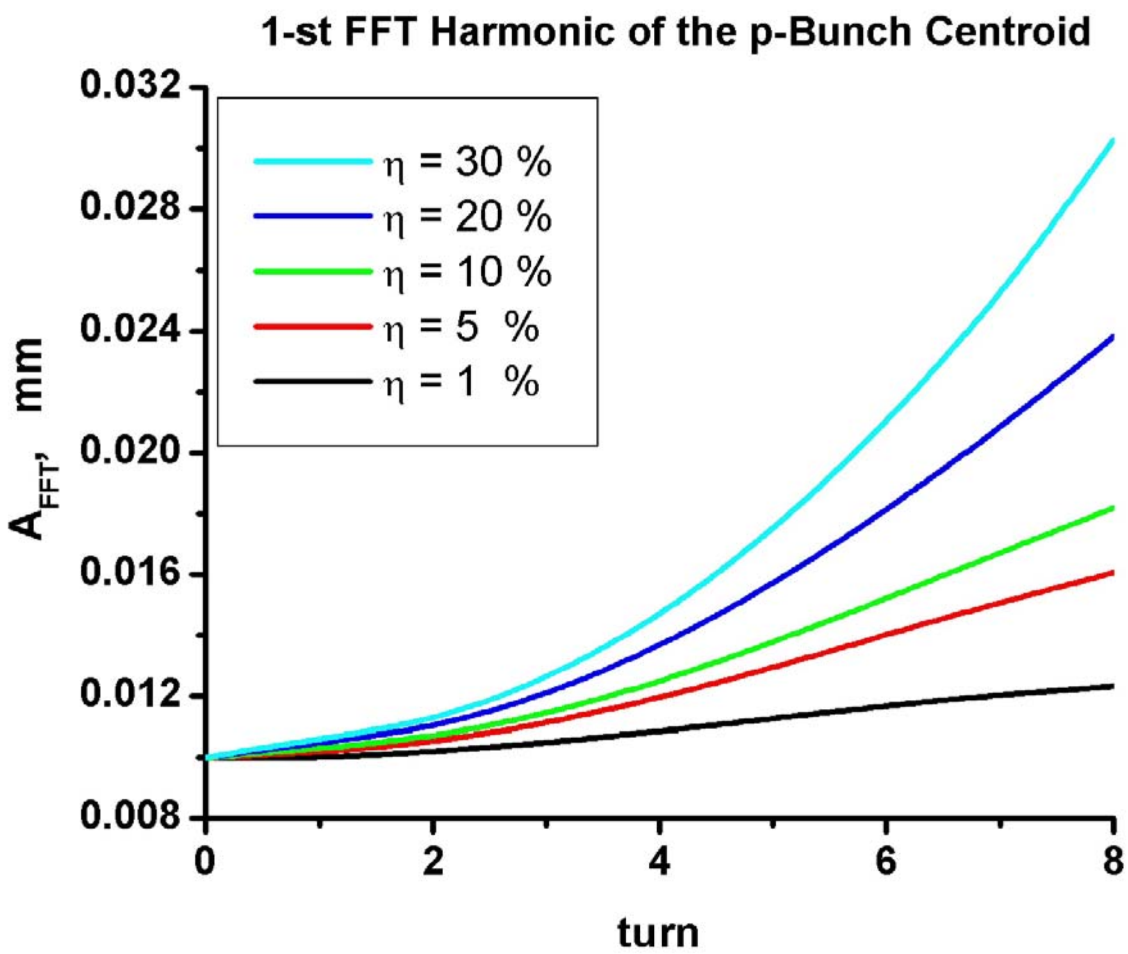




10 turns in the periodic structure requires about 10 min in SNS 16 CPUs

The growth of both electron and proton centroids matches for first several turns

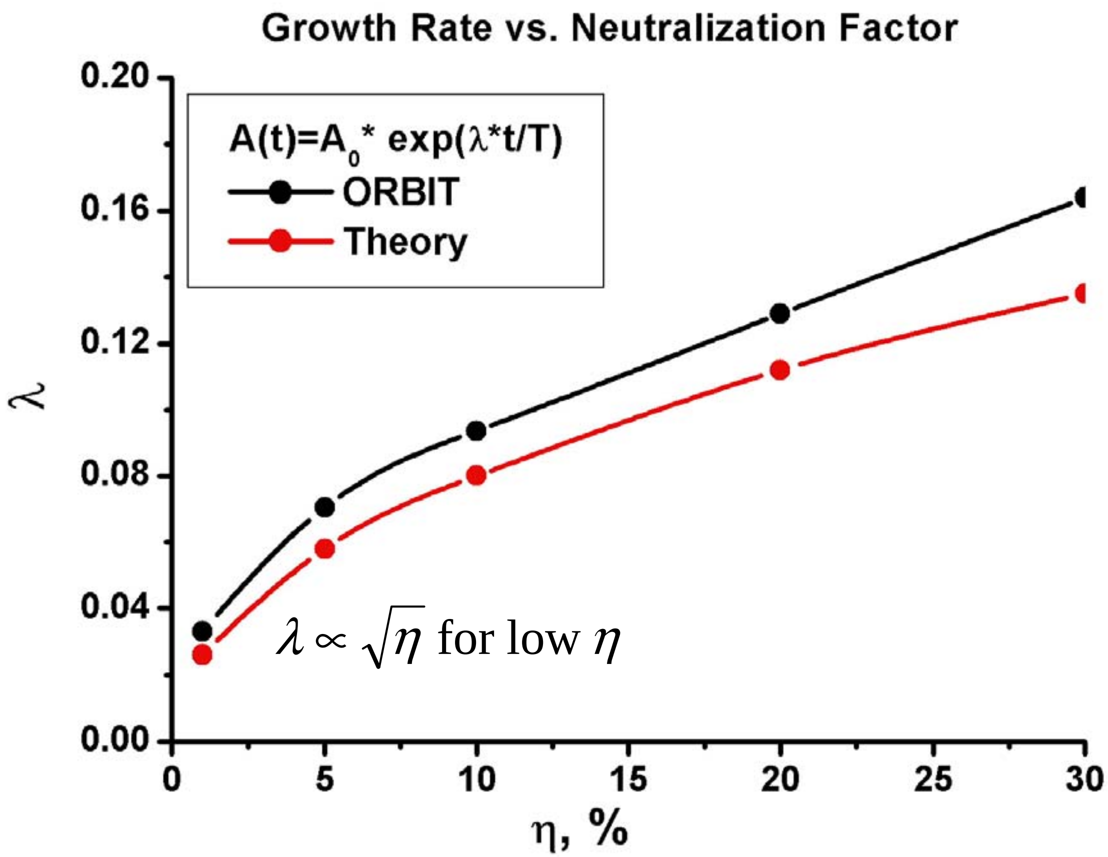
Two stream benchmark (ORBIT Simulation), cont.



The larger neutralization factor, the sooner e-cloud exceeds p-bunch radius.

We can apply the analytic two stream model for the first several turns

Two stream benchmark (ORBIT Simulation), cont.



The ORBIT growth rate is about 20% lager than the theory.

$$\frac{1}{\tau} \approx \frac{Q_p \omega_0}{2} \sqrt{\frac{Q_e}{|n - Q_e|}} \approx \frac{Q_p \omega_0}{2} \sqrt{\frac{Q_e}{Q_\beta}} \propto \sqrt{\eta}$$

Initial centroid modulation is for [Re=Rp=30mm]
However, we use Re=26mm to ensure linear force

Each proton spends outside of the e-cloud in some part of its trajectory

Estimation of computational requirements for PSR bunched beam case

Two stream model for PSR:

$$a_e = 12\text{mm},\ b_e = 15\text{mm},\ a_p = 16\text{mm},\ b_p = 20\text{mm},\ \ 0.793\,\text{GeV protonbeam}$$

$$\lambda_p = \tfrac{1.0\times 10^{14}}{90.261\text{m}} = 1.108\times 10^{12}\,[\text{m}^{-1}],\quad \text{betatron tune } Q_x = 3.21,\quad Q_y = 2.19$$

$$Q_{e,x} = 79.516,\quad Q_{e,y} = 71.121,\quad Q_{p,x} = 1.82\sqrt{\eta},\quad Q_{p,y} = 1.63\sqrt{\eta}\quad ;\eta = \lambda_e / \lambda_p$$

$$\boxed{\text{most unstabale at}\quad n_x = 83,\quad n_y = 73}$$

For PSR bunched beam we need to think:

- About 80*20 longitudinal nodes to simulate the PSR ring

- The simple case, no boundary, no 3D proton-proton space charge and no longitudinal momentum spread, will require about 80 times as much CPU time as our benchmark calculation (80 min. for 1 turn with SNS 16 CPUs)

- Setting primary electron production and secondary emission surface instead of linear neutralization factor

Conclusion

A benchmark of the code with an analytic model for two stream instabilities has been successfully done.

We are going to simulate a PSR bunched beam case.

Attachment for page 5 “32 points” symmetry